

MSE-422 – Advanced Metallurgy 2024/25

Exercise 6 – 11.12.2024 Thermodynamic Modeling & Metals Processing

1. Thermodynamic Modeling

A binary alloy system is modeled using a regular solution approach. The Gibbs free energy of mixing is:

$$\Delta G_{mix} = RT (x_A \ln x_A + x_B \ln x_B) + \Omega x_A x_B$$

- 1. Derive the condition for spinodal decomposition by finding the second derivative of ΔG_{mix} with respect to x_A . Express the spinodal curve as a relationship between \mathbf{x}_A and T.
- 2. For $\Omega = 20,000 \text{ J/mol}$ and $R = 8.314 \text{ J/mol} \cdot \text{K}$, calculate the critical temperature T_c , where spinodal decomposition occurs assuming $x_A = x_B = 0.5$:

$$T_c = \frac{\Omega}{2R}$$

3. At $T = T_c$, calculate the Gibbs free energy of mixing, ΔG_{mix} , for $x_A = 0.5$ and $x_B = 0.5$. Compare the ideal solution contribution to the regular solution term.

2. Sub-Regular Solution: Practical Example with Cu-Ni Alloy

The Gibbs free energy of mixing for a binary Cu-Ni system is modeled using the sub-regular solution model:

$$\Delta G_{mix} = RT \left(x_{Cu} \ln x_{Cu} + x_{Ni} \ln x_{Ni} \right) + x_{Cu} x_{Ni} \left(L_0 + L_1 (x_{Cu} - x_{Ni}) \right)$$

where:

- $R = 8.314 \, \text{J/mol} \cdot \text{K}$
- $T = 1000 \,\mathrm{K}$
- $L_0 = -5000 \,\mathrm{J/mol}$,
- $L_1 = 2000 \, \text{J/mol}$,
- $x_{Ni} = 1 x_{Cu}$.
- 1. For compositions $x_{Cu} = 0.2, 0.5, 0.8$, compute the Gibbs free energy of mixing:

$$\Delta G_{mix} = RT (x_{Cu} \ln x_{Cu} + x_{Ni} \ln x_{Ni}) + x_{Cu} x_{Ni} (L_0 + L_1 (x_{Cu} - x_{Ni}))$$

Compare the contributions of the ideal solution term $(RT(x_{Cu} \ln x_{Cu} + x_{Ni} \ln x_{Ni}))$ and the interaction parameter term $(x_{Cu}x_{Ni}(L_0 + L_1(x_{Cu} - x_{Ni})))$ to ΔG_{mix} .

2. Phase Stability Analysis: A phase is stable if small fluctuations in composition do not cause a decrease in the total Gibbs free energy. Mathematically, stability requires:

$$\frac{\partial^2(\Delta G_{mix})}{\partial x_{Cu}^2} > 0$$

If $\frac{\partial^2(\Delta G_{mix})}{\partial x_{Cu}^2} < 0$, the system is unstable, and phase separation occurs.

Based on the provided values of $L_0 = -5000 \,\text{J/mol}$ and $L_1 = 2000 \,\text{J/mol}$, identify regions of phase stability and instability for the given compositions ($x_{Cu} = 0.2, 0.5, 0.8$).

3. Discuss how the signs and magnitudes of L_0 and L_1 affect the phase diagram and the miscibility gap.

3. Ti Making Process

Considering the low productivity and high cost of the Kroll process for Ti production, many researchers around the world are attempting to develop a new Ti melting process in which metallic Ti of the requested specification can be produced with high productivity and low cost. One representative process under development is the preform reduction process (PRP). High-purity metallic Ti powder (up to 99% pure) is produced by reduction with metal vapor.

- 1. Based on the Ellingham diagram, define possible metals that can be used for the reduction of TiO₂.
- 2. Select the most appropriate reductant and explain why.
- 3. Define the overall reaction of the reduction process.

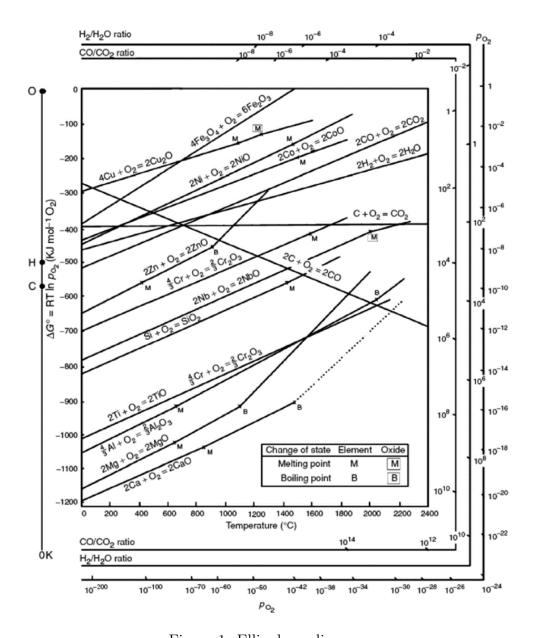


Figure 1: Ellingham diagram

4. Iron Making: The Blast Furnace

In a furnace, iron ore is reduced according to the following reaction:

$$Fe_2O_3 + 3CO \rightarrow 2Fe + 3CO_2$$

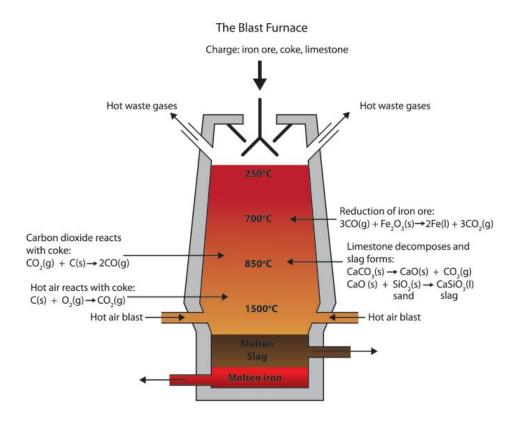


Figure 2: Blast furnace

Coke with a composition of 94% C is used to produce CO by combustion with air at the bottom of the furnace. Of the coke charged, 90.5% burns to CO only.

- 1. Define the input and the output of the process.
- 2. Define the function of coke charged in the furnace.
- 3. Define the combustion reaction to produce CO.
- 4. Calculate the volume of CO required to produce 1000 kg of iron.
- 5. Calculate the weight of coke required to produce 1000 kg of iron.

5. Oxygen Converter Process

For a BOF heat, the following data are given:

Hot metal	1% Si	0.15% P	0.25% Mn	3.5% C	
Slag	54% CaO	18% FeO	2.5% MgO	2.5% MnO	$CaO/SiO_2 = 3.2$

Table 1: Composition of the pig iron and slag

The weight of the scrap is 10% of the hot metal, and the steel at the tap contains 0.2% C.

- 1. Name the reaction equations and oxidation products of the process.
- 2. Explain why lime is normally added during the converter process.
- 3. Calculate, per ton of steel, the weight of the hot metal charge (assume scrap as pure Fe).
- 4. Calculate, per ton of steel, the weight of slag produced.
- 5. Calculate, per ton of steel, the quantity of lime required.

6. Environmental Aspects of Steel Production

In the last years, wind power has been considered a regenerative, CO₂-emission-free energy source.

Wind power plants:

- Direct emissions: 0
- Indirect emissions from the manufacturing and installation (use of steel):
 - Amount of CO₂ per ton of steel produced: 1500 kg/ton
 - Amount of steel per wind capacity: 400 ton/1 MW

Coal power plants:

- Direct emissions: $0.8 \,\mathrm{kg} \,\mathrm{CO}_2/1 \,\mathrm{kWh}$ of electricity generated
- 1. Considering a wind power plant of 1 MW, how much energy can be produced by a coal power plant emitting the same amount of CO2?
- 2. Calculate the minimum operating time that the installed wind power capacity has to run to offset the CO₂ emitted during steel production compared to conventional electricity production via hard coal power plants.

Note: The solutions to this exercise will be available on Moodle from the next exercise session.

Contacts:

- Jian Yang (jian.yang@epfl.ch)— responsible for this exercise
- Seyyed Ezzatollah Moosavi (ezzatollah.moosavi@epfl.ch)